We welcome Dan Levin’s comment as a nice example of the manner in which we believe science should ideally proceed. The story begins with a real-world observation: the existence of multi-unit uniform-price auctions. It continues with a theory: Engelbrecht-Wiggans and Kahn’s (1998) theory of demand reduction in uniform-price auctions.\(^1\) Next came an experiment: List and Lucking-Reiley’s (2000) field experiment at a sportscard show, which tests the predictions of that particular theory. The experiment confirmed some of the theory’s predictions, but contradicted one prediction in particular. Now comes a new theory that proposes to explain the anomaly discovered in the experiment. Even more appealing, the theory makes new testable predictions not contained in the previous theory or experiments. We wish to highlight this important point because we feel that examples of this sort of scientific progress are all too rare in economics. We encourage other economists to follow Levin’s example in responding to anomalous empirical findings with new theories that contain new testable implications.

Levin proposes an explanation for the behavior observed by List and Lucking-Reiley (2000) in two-bidder, two-unit auctions. While Engelbrecht-Wiggans and Kahn (1998, which we shall denote by “EWK”\(^2\)) predict first-unit bids to be equal across the uniform-price and multi-unit-Vickrey auction formats, List and Lucking-Reiley find instead that first-unit bids are significantly (both statistically and economically) higher in the uniform-price format than in the Vickrey format.\(^3\) Levin identifies another Nash equilibrium (BSP*) for the uniform-price auction, one which List and Lucking-Reiley had ignored, where bidders bid well above value on first-unit bids and zero on second-

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2 There are several important theory articles on the topic of uniform-price auctions, dating from Vickrey’s classic 1961 paper. Levin’s comment refers to the theoretical propositions on demand reduction in an unpublished paper by Ausubel and Cramton (2002); List and Lucking-Reiley test propositions from this article as well as from the published article by Engelbrecht-Wiggans and Kahn (1998). Other related theoretical articles include those by, Noussair (1995) and Katzman (1999).

3 Levin calls this equilibrium “LLRE,” after List and Lucking-Reiley, but those two authors prefer to use the notation EWK. List and Lucking-Reiley presented no new theory in their article, and based most of their experimental tests on the theory of \(n\) private-value bidders bidding for 2 units, laid out in detail by EWK, with some tests also derived from Ausubel and Cramton (2002).

4 We remind the reader of the difference between the two formats for the concrete case where there are two units being auctioned. In a uniform-price auction, the highest two bids each win one unit at a price equal to the highest-rejected (third-highest) bid. In a Vickrey auction, the highest two bids each win at prices equal to the bids they displaced – with two bidders, by definition this means the prices equal the amounts of the third-highest and fourth-highest bids.
unit bids. A high first-unit bid effectively helps to “enforce” a demand-reduction equilibrium, making it impossible for a rival to win a second unit at a profitable price, and therefore giving the rival an incentive to bid zero on the second unit to keep the price down on the first unit she wins. Bidders therefore earn relatively high profits in this equilibrium, as each bidder wins one unit at a price of zero. Levin notes several additional advantages of his equilibrium compared with the EWK equilibrium.

We wish to raise three concerns with Levin’s proposal. First, Levin fails to consider whether his Nash equilibrium holds in the multi-unit Vickrey auction as well as in the uniform-price auction. In order to explain the experimentally observed, his theory should predict differences in equilibrium bidding behavior across the two auction formats. To address this, we will evaluate the existence and other properties of his equilibrium in the Vickrey auction format. Second, we temper Levin’s claims that his equilibrium generally provides higher efficiency and higher bidders’ surplus than the EWK equilibrium does. Third, we show that given the empirical distribution of bids in the experiment, a bidder has a good reason not to follow Levin’s prescription of bidding above value on the first unit.

I. Why BSP* in the Uniform-Price but not the Vickrey Auction?

First, we examine to what extent Levin’s BSP* equilibrium can be applied to the multi-unit Vickrey auction, a question Levin has not considered. To explain the difference in behavior across auction formats, the theory must tell us why first-unit bids are higher in the uniform-price auction than the Vickrey auction. In stating that empirical bids are “above value” for the uniform-price auction, Levin implicitly seems to assume by contrast that Vickrey-auction bids are approximately equal to values (the dominant strategy). This is not necessarily the case: because this was a field experiment rather than a laboratory experiment, we do not observe bidders’ demand curves. Indeed, in his work with John Kagel, Levin has been a leader in documenting that bidders tend to overbid relative to value in Vickrey auctions (see Kagel and Levin (1993), Kagel and Levin (2001)). Two recent studies directly replicate the results of List and Lucking-Reiley (2000) in the laboratory with two bidders and two goods, and both find evidence that bidders tend to bid more than value on the first unit in both the Vickrey and the uniform-price format. Porter and Vragov (2003) induce declining demands for each bidder, while Engelmann and Grimm (2002) induce flat demands for each bidder. Consistent with the results of List and Lucking-Reiley (2000), both studies find a difference between auction formats in first-unit bids: bidders overbid more in the uniform-price than in the Vickrey format.

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4 Engelbrecht-Wiggans and Kahn (1998) ruled out this equilibrium because it employs weakly dominated strategies. Levin argues, however, that the other advantages of his equilibrium (relative to EWK) outweigh this disadvantage.
5 In some cases, there may be multiple EWK equilibria. For simplicity, our discussion assumes there is only one. Yet, each of our statements about “the EWK equilibrium” apply to all EWK equilibria.
6 Kagel (1995) reviews the extensive laboratory evidence on overbidding in single-unit Vickrey auctions, where it is a weakly dominant strategy to truthfully reveal one’s value. The multi-unit-auction laboratory results are therefore consistent with the single-unit-auction laboratory results. Since we cannot observe values directly in the field experiment, we concern ourselves primarily with explaining bidding differences between uniform-price and Vickrey auctions.
Unfortunately for Levin, it turns out that BSP* is also an equilibrium to the Vickrey auction, casting some doubt on his theory’s ability to explain the difference in bidding behavior between the uniform-price and Vickrey formats. To see that BSP* is an equilibrium in the Vickrey auction, suppose that my rival is submitting a bid of $\beta \geq 1$ on the first unit and a bid of zero on the second unit. If I adopt the same strategy, then we each win one unit at a price of zero. Can I do any better by adopting a different strategy? If I set the amount of my higher bid to any value above zero, and keep my lower bid below the other bidder’s bid $\beta$, I still win one unit at a price of zero, and do not change my surplus. If I set both bids equal to zero, I risk winning nothing and earning zero surplus, so this makes me worse off. The final case is that I set both bids greater than or equal to $\beta$; in this case I win two units instead of one, but I pay the amount $\beta$ for each unit and therefore earn a negative surplus. Thus, there is no gain to deviating from BSP*, and therefore BSP* is an equilibrium to the multi-unit Vickrey auction.

In order for Levin’s theory to explain the data, we must therefore provide a theoretical reason why BSP* should be played in the uniform-price auction but not in the Vickrey auction. Levin notes several advantages of BSP* relative to EWK in the uniform-price auction, including: (1) BSP* bid levels are easy to choose, as they do not depend on the shape of the distribution of possible bidder values, (2) BSP* has no bidder regret in equilibrium, and (3) bidders share the surplus more equally in BSP*. We might ask whether BSP* has these same advantages relative to truth-telling in the Vickrey auction. It turns out that truth-telling involves strategies just as easy as BSP* and yields no ex-post regret, but BSP* does divide surplus more equally among the bidders than truth-telling does. Thus, only one of BSP*’s three relative advantages in the uniform-price auction also occurs in the Vickrey auction. We personally find the relatively low cognitive cost of formulating a bidding strategy to be the most attractive advantage of BSP* relative to EWK. Since this relative advantage of BSP* disappears in the Vickrey auction, we

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7 We believe Levin uses the term “regret” in the same sense used by Engelbrecht-Wiggans (1989), the first paper we know of to have examined regret in the context of bidding. Consider an example where EWK has ex-post regret while BSP* does not—a 2-bidder, 2-unit auction with minimum bid equal to zero, and suppose each bidder obtains her values from two independent draws from a uniform distribution on [0,1]. As shown by EWK in their Example 1, the unique EWK equilibrium in pure strategies is for each bidder to bid truthfully on the first unit and zero on the second unit, so each bidder wins one unit at a price of zero. Now consider the realization of values where Bidder A values the two units at 0.9 and 0.8 respectively, while Bidder B values the two units at 0.3 and 0.2, respectively. In equilibrium, Bidder A earns a surplus of 0.9. After observing B’s bids, A would wish that she had submitted a second-unit bid of 0.31 instead of 0, thus beating Bidder B’s first-unit bid, and therefore winning two units for a total surplus of 1.1 instead of winning a single unit for a surplus of 0.9. Thus, Bidder A has regret in this particular example. By contrast, in BSP*, she could not increase her ex-post surplus by increasing her second-unit bid because she could only win a second unit by submitting two bids greater than or equal to 1 (exceeding Bidder B’s first-unit bid), in which case she earns a total surplus of $-0.3$ instead of $+0.9$.

8 Levin actually claims five advantages of BSP* relative to EWK, but we are less sanguine about the last two. The fourth claim, that efficiency is at least as high in BSP* as in EWK, turns out to be false (see below). The fifth claim, that total bidders’ surplus is often higher in BSP* than in EWK, seems overstated. Below, we give one example of realized bidder values where surplus is higher in BSP*, and another where surplus is higher in EWK. Since we do not know whether BSP* provides a Pareto improvement on average (over all possible realizations of bidder values), we are reluctant to agree that increased bidder surplus is a clear advantage of BSP* over EWK.

9 Bidding equal to one’s value does not involve any complicated calculations involving the distribution of others’ values, and since truth-telling is a weakly dominant strategy in the Vickrey auction it cannot involve regret.

10 When bidders play the truth-telling equilibrium, one bidder can win both units. This cannot occur in BSP*.

11 BSP* also involves lower bidder payments than truth-telling (zero versus positive amounts), but total bidders’ surplus is not necessarily higher because BSP* involves lower efficiency. See below.
conclude that there is at least one good reason why BSP* might be relevant in the uniform-price auction but not in the Vickrey auction.

II. Does BSP* Really Provide Higher Payoffs to Bidders?

Second, we examine the question of whether Levin’s BSP* equilibrium provides higher efficiency and increased payoffs for the bidders. Levin asserts that “it is not the case that efficiency is higher in [EWK] than in BSP*. In fact, the reverse is often true.” He uses this assertion, combined with the assertion that equilibrium payments are always lower in BSP*, to conclude that “total bidders’ surplus is often larger in BSP* than in [EWK].” However, we can show that the efficiency assertion is not true; in fact, EWK provides weakly higher efficiency than BSP*.

To show this, we consider two cases. First, for some realizations of values, the EWK equilibrium may allocate one unit to each bidder, just as in Levin’s BSP* equilibrium. In this case, efficiency is the same in both equilibria. Second, the EWK equilibrium may allocate both units to the same bidder. This means that the winning bidder’s second bid is higher than the losing bidder’s first bid. Since first bids are truthful and the second bid is at most equal to the second value, this means that the winning bidder’s second value is at least as great as the losing bidder’s first value. This makes it efficient to allocate both units to the same bidder, as in EWK, and inefficient to split the units between the two bidders, as in BSP*. So Levin’s equilibrium is never more efficient than the EWK equilibrium, and sometimes less efficient.

Even though BSP* is in general less efficient than EWK, bidders might still be jointly better off under BSP*. There are three cases to consider. In the first case, the EWK equilibrium involves each bidder bidding zero on the second unit, so the allocations and bidder surplus are the same as in BSP*. In the second case, EWK has at least one bidder submitting a nonzero second-unit bid, and the two second-unit bids turn out to be lower than the two first-unit bids. In this case, the bidders split the two units, each paying a positive amount (the third-highest bid), so EWK represents the same allocation as BSP* with higher prices, which means the bidders are better off under BSP*. In the third case, EWK has at least one bidder submitting a positive second-unit bid, and this bidder ends up winning both units. Then efficiency is higher in EWK than in BSP*, but it involves higher (nonzero) payments to the seller. The increased efficiency may or may not outweigh the higher payments, making bidders better off in EWK than in BSP*. One might conjecture that the expected bidder surplus (over all possible realizations of values) is always higher under BSP* than under EWK, in which case BSP* would provide a Pareto improvement, but no one has yet proven this statement to our knowledge. While

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12 We also disagree slightly with the assertion about bidder payments. Equilibrium payments are not “always lower in BSP*” than in EWK, at least not in a strict sense. For some distributions of values, EWK predicts second-unit bids equal to zero, so bidder payments in both equilibria are equal to zero, and thus equal to each other (not “always lower in BSP*”). However, Levin’s statement is true if expressed as a weak rather than strict inequality: bidder payments in BSP* are always less than or equal to bidder payments in EWK.
13 Assuming that the winner’s second bid is not exactly equal to the loser’s first bid, an event which happens with probability zero for continuous probability distributions of values.
14 By EWK’s Corollary 4.4, second-unit bids can equal zero in equilibrium whenever the marginal density of $v_{1,t}$ is nondecreasing.
Levin claims that total bidders’ surplus is *often* larger in BSP* than in [EWK],” we feel that “sometimes” would be more accurate than “often.”

### III. Beliefs inconsistent with the actual data

Finally, Levin’s theory requires bidders to have beliefs\(^{15}\) that are inconsistent with the actual data. To understand Levin’s implicit assumption about beliefs, consider an auction with 2 units, 2 bidders, and a minimum bid of zero. Suppose that bidder A has values of 0.5 and 0.4, and that bidder B has values of 0.9 and 0.7. The BSP* equilibrium then prescribes that each bidder should bid 1 on the first unit\(^{16}\) and 0 on the second unit; then each bidder wins one unit at a price of zero, and bidder A obtains a surplus of +0.5. Suppose instead that bidder B does not understand the equilibrium he is supposed to be playing, and he instead submits bids of 0.9 and 0.6. Then each bidder again wins one unit, but now the highest rejected bid of 0.6 determines the price, and therefore bidder A obtains a surplus of −0.1. This means that bidder A would have been better off following the EWK strategy of bidding truthfully (0.5) on the first unit, earning a surplus of zero, instead of following the BSP* strategy and earning a negative profit. No matter what strategy B might pursue and no matter what A’s second-unit bid might be, A can never do better on her first-unit bid than to bid equal to her first-unit value. In general, bidder A would be strictly worse off following BSP* instead of EWK if there is even the slightest chance that bidder B might submit a second-unit bid higher than A’s first-unit value.

Levin’s equilibrium thus depends on each bidder believing that her rival will never submit a second-unit bid greater than her own first-unit value, but it turns out that such a belief is inconsistent with the experimental data on actual bids. Indeed, List and Lucking-Reiley (2000) report that nearly three-fourths of all second bids were strictly positive. They also report that the average second bid in the uniform-price auction ranged, depending on the treatment, from about 30 percent to about 60 percent of the average first bid in the Vickrey auction. Similarly, the laboratory experiments of Engelmann and Grimm (2003) and Porter and Vragov (2003) both observe a substantial fraction of second-unit bids to be higher than the median first-unit value in the distribution. Experimental observations thus demonstrate a very real chance that a bidder would find her competitor making two bids both in excess of her first-unit value.\(^{17}\)

### IV. Concluding Remarks

\(^{15}\) Here, we use “beliefs” not in the technical sense of beliefs about other players’ uncertain types, but rather to conjectures the players are making about each others’ strategies in a situation with multiple equilibria.

\(^{16}\) Levin notes that a first-unit bid of any amount greater than or equal to 1 can be a BSP* equilibrium. In this example, we assume a first-unit bid equal to 1 for simplicity, but our caveat holds even more strongly for first-unit bids above 1.

\(^{17}\) A related disadvantage of BSP* is that it predicts all second-unit bids to be exactly zero, but this is clearly not true in the data. List and Lucking-Reiley found that 80% of their second-unit bids were nonzero. We do not wish to emphasize this observation, because no economic theory ever predicts behavior exactly. For example, Levin’s theory might be approximately true (bidders do not bid exactly zero even though they are motivated to make one very high and one very low bid). Also, some fraction of bidders may be playing BSP* (or approximately BSP*) while some other fraction are playing EWK in the uniform-price auction. Provided the fraction playing BSP* is lower in the Vickrey auction than in the uniform-price auction, this would be enough to produce the experimental results.
Despite our caveats, Levin’s model is the only theory we have seen that can explain the experimental observation of first-unit bids being higher in uniform-price auctions than in Vickrey auctions. An even greater strength of Levin’s theory is the fact that it makes two new predictions. First, this equilibrium disappears when the number of bidders increases beyond two; with three or more bidders we are left with only the EWK equilibrium, so first-unit bids should be equal across auction formats. Second, this equilibrium also disappears when the seller implements a binding reserve price, setting the minimum bid high enough to have positive probability of exceeding a given bidder’s willingness to pay for a second unit of the good. Levin notes that in our more recent work, we have already confirmed the first of these two predictions. It will be interesting to see whether the second prediction is correct as well. To Levin’s credit, he made these two predictions before he ever saw our data. We find this very exciting, as we feel economic science advances more quickly when theorists go out on a limb to make new testable predictions.
References


